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On the non-stationary evaporation kinetics. I. Mathematical model and experimental data

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Abstract

A theoretical analysis of the non-stationary evaporation kinetics based on experimental data has been done. A comparison between non-stationary diffusion rate and convective mass flow as a result of a Stephan flow is proposed. It is shown, that the evaporation do not change the temperature of the liquid interface. The mass transfer rate as a result of a diffusion and convection differs from the evaporation velocity, which is easy to explain with a natural convection on account of a instability of the system.

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1. Introduction

The problems of the mass and heat transfer mechanisms and kinetics in systems with intensive interphase mass transfer are objects of communications appeared [1]. Upon such conditions secondary flows occur at the phase interface that lead to several non-linear effects changing both the mechanism and the kinetics of the simultaneous mass and heat transfer processes as well as the hydrodynamic stability. As a result of the above mentioned effects a lot of experimental data cannot be predicted by the linear theory [2]. The disagreements between the experiments and the linear theory have been accepted as contributions of the Marangoni effect. However, in some cases they may be considered as non-linear mass transfer effects induced by large concentration gradients [1].

An opportunity for a comparative analysis of the Marangoni effect (a secondary flow induced by the surface tension gradient) and the effects of the non-linear mass transfer (secondary flow due to large concentration

gradient) is the mass transfer upon non-stationary absorption (desorption) in stagnant gas–liquid system [3].

The investigations on the absorption and desorption of low soluble gases [3–5] upon conditions of a non-stationary mass transfer between stationary gas and liquid layers have demonstrated discrepancies between the rates of the absorption and the desorption processes.

It was demonstrated that the desorption satisfies the linear theory, while the absorption rate is significantly greater. It is interesting to note that at small times ($t < t_{cr}$) both rates are equal and satisfy the linear theory. However, at $t > t_{cr}$ the absorption rate increase significantly. The attempts to explain the fact [3–5] and the assumption that at $t < t_{cr}$ the process is stable, while at $t > t_{cr}$ it becomes unstable are hard acceptable because there are no a physical hypothesis (a mechanism) and a corresponding mathematical model, where the time (t_{cr}) is a parameter and the stability depends on this parameter.

Our investigations [6–9] indicate that the rate of the non-stationary absorption J_a ($\text{kg}/\text{m}^2 \text{ s}$) defined by the relationship:

$$J_a = c^* \left[\left(\frac{D}{\pi t_0} \right)^{1/2} + \frac{5}{4} \gamma \left(\frac{g}{v} \right)^{1/2} D^{3/4} t_0^{5/4} \right], \quad (1)$$

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Nomenclature

a	thermal diffusivity of liquid (m ² /s)	t	time (s)
c	concentration in volume (kg/m ³)	v_s	velocity of Stephan flow (m/s)
c^*	concentration at interface (kg/m ³)	v	velocity of the flow (m/s)
D	diffusivity (m ² /s)	z	coordinate (m)
g	gravity (m ² /s)	λ	conductivity (kcal/m s °C)
J	evaporation rate (kg/m ² s)	ϑ	kinematic viscosity (m ² /s)
q	heat effect of evaporation (kcal/kg)	π	3.14
Q	account of the evaporation liquid (kg/m ²)	ρ	density (kg/m ³)

where the first term represents the rate of the main process (the stable non-stationary absorption) and the second term contributes the effect of the increased absorption rate as a result of the loss of stability. It follows explicitly from (1) that at small time the second term is so much smaller than the first term, because the effect of the stability disturbance depends on the time and it occurs during sufficiently large values of t_0 .

The attempts to explain the effects concerned here by the contribution of the Marangoni effect [3–5,10] should not reach a success since the thermal effect upon the condition of the absorption of low soluble gases is not able to create temperature gradients at the liquid surface, i.e. there is not surface tension gradient.

In the cases where the liquid surface is heated [12] with a sufficient intensity the Marangoni effect occurs within a thin layer of the liquid phase ($2\text{--}3 \times 10^{-3}$ m). This thickness is significantly lower than that employed in [3–5].

The theoretical analysis in [11] suggests a simultaneous existence of both the volumetric and surface thermal source, the stationary process occurs at small times and the occurrence of species (as a result of the absorption) having properties of surfactants. Obviously, it is hard to accept that such systems are of practical interest, where these conditions are executed.

The studies commented above are addressed to the absorption (desorption) of low soluble gases when the liquid mass transfer resistance limits the mass transfer rates. Obviously, it is interesting to focus the investigations on situation where the mass transfer is limited by the gas phase. An adequate example is the case of non-stationary evaporation of a stagnant liquid layer in a stagnant gas phase above it. Such systems are investigated experimentally in details in [13].

2. Physical model

The non-stationary evaporation of a liquid with a moderate partial pressure (water, methanol, ethanol and *i*-propanol) at 20 °C in an inert gas (nitrogen, argon and helium) will be investigated. The process occurs in a

thermostatic condition, corresponding to the experiments performed in [13]. Upon such conditions the non-stationary mass transfer of the liquid vapour in the gas phase limits the process rate.

The mechanism of the non-stationary evaporation may be considered as a non-stationary diffusion complicated with additional effects of a variable temperature at the liquid surface (as a result of the thermal effect of the evaporation phenomenon) and a convection (secondary Stephan flow) as well as a natural convection. The contribution of these phenomena on the evaporation rate will be analyzed consequently.

3. Interface temperature effect

The investigations [13] on the evaporation rates of liquids (H₂O, CH₃OH, C₂H₅OH, *i*-C₃H₇OH) shows a time dependent average liquid temperature. In fact, the process depends on the surface temperature (θ^* , °C) only.

The temperature distribution in a layer of an evaporating liquid is described by

$$\frac{\partial \theta}{\partial t} = a \frac{\partial^2 \theta}{\partial z^2};$$

$$t = 0, \quad \theta = \theta_0; \quad z = 0, \quad \lambda \frac{\partial \theta}{\partial z} = qJ; \quad z = h, \quad \theta = \theta_0,$$
(2)

where the coordinate axis z is oriented normally to the liquid–gas interface ($z = 0$), θ and θ_0 (°C)—temperature of the liquid and her initial value; t (s)—time; λ (kcal/m s °C)—the thermal conductivity of the liquid; a (m²/s)—the temperature diffusivity; q (kcal/kg)—the latent heat of the evaporation; J (kg/m² s)—the evaporation rate; h (m)—the thickness of the evaporating liquid.

The evaporation rate J in Eq. (2) may be determined through experimental data concerning the amount of the evaporated liquid Q (kg/m²) at a time t (s). Such data are available in [13]. After a sufficiently large initial time interval (where J has greatest values) the relationship may be expressed as

Table 1
Characteristic parameters of the gas–liquid system (20 °C)

Parameters	Systems					
	H ₂ O/N ₂	H ₂ O/He	H ₂ O/Ar	CH ₃ OH/Ar	C ₂ H ₅ OH/Ar	<i>i</i> -C ₃ H ₇ OH/Ar
q , kcal/kg	584.3	584.3	584.3	280.0	217.9	179.1
$a \times 10^7$, m ² /s	1.43	1.43	1.43	1.05	0.888	0.752
$\lambda \times 10^5$, kcal/m s °C	1.448	1.448	1.448	4.875	4.015	3.657
$D \times 10^5$, m ² /s	2.41	2.57	8.86	0.98	1.0	0.846
$\theta_0 - \theta^*$, °C	0.3	0.3	0.2	3.4	0.6	0.4
$A_e \times 10^4$, kg/m ² s ^{1/2}	2.051	2.190	1.633	19.47	3.963	3.287
$A_D \times 10^4$, kg/m ² s ^{1/2}	0.936	0.967	1.79	5.09	3.89	1.51
$A \times 10^4$, kg/m ² s ^{1/2}	0.954	0.961	1.94	5.92	4.28	1.57
c^* , kg/m ³	0.0142	0.0143	0.0161	0.142	0.0995	0.0939
c_0^* , kg/m ³	1.13	1.66	0.162	1.48	1.57	1.63
$\vartheta \times 10^5$, m ² /s	1.441	1.360	12.12	1.360	1.360	1.360
ρ_0 , kg/m ³	1.16	1.66	0.166	1.66	1.66	1.66
$Q_{\max} \times 10^2$, kg/m ²	0.3660	0.3684	0.4129	3.644	2.558	2.414
α	-0.555	0.778	-1.216	-0.246	0.133	0.335

$$Q_e = A_e \sqrt{t}, \quad (3)$$

where A_e (kg/m² s^{1/2}) may be determined on the basis of experimental data reported in [13] for systems such as: H₂O/N₂, H₂O/He, H₂O/Ar, CH₃OH/Ar, C₂H₅OH/Ar and *i*-C₃H₇OH/Ar (see Table 1). This allows to define (by means of Eq. (3)) the values of J_e as

$$J_e = \frac{dQ_e}{dt} = \frac{A_e}{2\sqrt{t}}. \quad (4)$$

The substitution of (4) into (2) permits to define [16] the temperature distribution within the evaporating liquid layer with a thickness h :

$$\theta = \theta_0 - \frac{qA_e\sqrt{\pi a}}{2\lambda} \sum_{n=0}^{\infty} (-1)^n \left[\operatorname{erfc} \frac{2nh+z}{2\sqrt{at}} - \operatorname{erfc} \frac{2(n+1)h-z}{2\sqrt{at}} \right]. \quad (5)$$

Equation (5) allows to determine the temperature variations at the liquid top surface ($z = 0$) as a result of the evaporation process:

$$\theta_0 - \theta^* = \frac{qA_e\sqrt{\pi a}}{2\lambda} \left[1 + 2 \sum_{n=1}^{\infty} (-1)^n \operatorname{erfc} \frac{nh}{\sqrt{at}} \right], \quad (6)$$

where the liquid thickness was assumed $h = 3 \times 10^{-3}$ m.

It follows from (6) that the maximum of the temperature at the interface may be reached at the limiting situations of $t \rightarrow 0$ or $h \rightarrow 0$.

$$\theta_0 - \theta^* = \frac{qA_e\sqrt{\pi a}}{2\lambda}. \quad (7)$$

The result (7) could be obtained also in the cases when the liquid layer is not upon thermostatic conditions. In such situation the last boundary condition in (2) becomes $z \rightarrow \infty$, $\theta = \theta_0$ ($z > h_a = \sqrt{at}$), where h_a is the

thickness of thermal boundary layer. Thus the temperature distribution is

$$\theta = \theta_0 - \frac{qA_e\sqrt{\pi a}}{2\lambda} \operatorname{erfc} \frac{z}{2\sqrt{at}}. \quad (8)$$

Equation (8) shows that the results (7) follows directly at $z = 0$.

The results concerning the interface temperature $\theta - \theta_0$ are listed in Table 1. It is clear that the temperature at the liquid surface practically remains unchanged by the evaporation process. Some deviations from that “rule” are demonstrated by the system CH₃OH/Ar, but they change the partial pressure of the vapours above the liquid in the range of 10% that should be neglected (the data of [13] have the same experimental error).

The experimental dependence $\theta(\sqrt{t})$ (obtained in [13]) shows that the asymptotic value $Q = Q_{\max}$ is reached at large values of \sqrt{t} . It allows to determination of exactly the vapour concentration value on the interface $c^* = Q_{\max}/h$ (kg/m³) and the results are shown in Table 1. For instance, the results present that the CH₃OH surface temperature has been 15–16 °C.

4. Diffusion rate effect

The experimental data concerning θ obtained in [13] allow determining the value A_e in (3) and the results are summarized in Table 1. On the other hand, if the evaporation rate is limited by the non-stationary diffusion, the concentration distribution is [6–9]:

$$c = c^* \operatorname{erfc} \frac{z}{2\sqrt{Dt}}, \quad (9)$$

where c and c^* (kg/m^3) are the concentrations of the vapours in the gas phase and at the interface respectively, D (m^2/s)—the diffusivity.

The solution (9) permits to obtain the rate of the diffusion

$$J_D = -D \left(\frac{\partial c}{\partial z} \right)_{z=0} = c^* \sqrt{\frac{D}{\pi t}} = \frac{A_D}{2\sqrt{t}}, \quad (10)$$

where

$$A_D = 2c^* \sqrt{\frac{D}{\pi}}. \quad (11)$$

The values of A_D are collected in Table 1. The comparison between the values of A_D and A_e indicates significant differences that may be attributed to the occurrence of a Stephan flow [15]. Some strange behaviors are demonstrated by the systems $\text{C}_2\text{H}_5\text{OH}/\text{Ar}$ ($A_e \approx A_D$) and $\text{H}_2\text{O}/\text{He}$ ($A_e < A_D$).

5. Effect of the Stephan flow

The difference between the evaporation rate and the rate of the non-stationary diffusion indicates that a convective contribution exists. The evaporation of a liquid in an inert gas is a results of a phase transition liquid vapours, so there is a volumetric effect of a heterogeneous reaction at the interface [15] that creates the Stephan flow. If the process occurs in a thermostatic conditions it is limited by both the diffusive and the convective transports in the gas phase. Thus the evaporation rate can be expressed as:

$$J = -D \left(\frac{\partial c}{\partial z} \right)_{z=0} + c^* v_s, \quad (12)$$

where v_s (m/s) is the velocity of the Stephan flow.

The mass transfer rate of the inert gas (in the gas phase) in the case of a binary system gas–vapour may be expressed in a similar manner:

$$J_0 = -D \left(\frac{\partial c_0}{\partial z} \right)_{z=0} + c_0^* v_s, \quad (13)$$

where c_0 and c_0^* (kg/m^3) are the concentration of the inert gas in the bulk of the gaseous phase and at the interface. If the evaporating liquid is saturated by the inert gas (like in the experiments performed in [13]) it follows that

$$J_0 = 0, \quad c_0 + c = \rho = \rho_0 + \alpha c, \\ \frac{\partial c_0}{\partial z} = -(1 - \alpha) \frac{\partial c}{\partial z}, \quad \alpha = \frac{\rho_V - \rho_0}{\rho_V}, \quad (14)$$

where ρ_V (kg/m^3) is the density of the vapour of the liquid, ρ_0 (kg/m^3)—density of the inert gas, while (kg/m^3)

ρ is the density of the gaseous phase. In this way is obtained from the Eqs. (12) and (13):

$$v_s = -\frac{D(1 - \alpha)}{c_0^*} \left(\frac{\partial c}{\partial z} \right)_{z=0}, \quad J = -D \frac{\rho_0}{c_0^*} \left(\frac{\partial c}{\partial z} \right)_{z=0}. \quad (15)$$

The comparison between the velocity of the Stephan flow (15) and the velocity of the secondary flow induced by the large concentration gradients [1] shows that they are different when there is a liquid evaporation in inert gases upon isothermal conditions (a thermostated system) since $\rho_0 \neq \rho^*$. Here

$$\rho^* = \rho_0 + \alpha c^*. \quad (16)$$

6. Effect of the convective transport

The convective mass transfer upon non-stationary evaporation from a stagnant liquid into a stagnant gas above it (within a large initial time interval) could be attributed to the Stephan flow and the natural convection. Let consider a gaseous layer above a stagnant liquid. The momentum equations of the gaseous phase and the convection–diffusion equations of the liquid vapours (upon the assumption of an one-dimensional approximation) are:

$$\frac{\partial v}{\partial t} + v \frac{\partial v}{\partial z} = \vartheta \frac{\partial^2 v}{\partial z^2} - \frac{1}{\rho_0} \frac{\partial p}{\partial z} - \frac{g\alpha c}{\rho_0}, \\ \frac{\partial c}{\partial t} + v \frac{\partial c}{\partial z} = D \frac{\partial^2 c}{\partial z^2}; \\ t = 0, \quad v = c = 0; \quad (17)$$

$$z = 0, \quad v = -\frac{D(1 - \alpha)}{c_0^*} \left(\frac{\partial c}{\partial z} \right)_{z=0}, \quad c = c^*;$$

$$z \rightarrow \infty, \quad v = c = 0,$$

where ϑ (m^2/s) is the kinematic viscosity.

The coordinate z is oriented vertically upwards and the liquid interface is $z = 0$. In the cases, when the Stephan flow does not exist (see (15)), its velocity is zero:

$$v(0, t) \equiv 0, \quad (18)$$

that leads to a stable solution of (17):

$$\bar{v} \equiv 0, \quad \bar{c} = a_1 z + a_2, \quad \frac{\partial \bar{p}}{\partial z} = g\alpha c, \quad (19)$$

i.e. the gaseous phase is stagnant, the concentration distribution is linear and the pressure gradient depends on the concentration distribution along the gaseous layer depth [14]. This is a stable state of the system, but small disturbances could lead to a new stable state, where the motion of the gaseous phase is a result of the natural convection.

It is possible to introduce in (17) the dimensionless variables

$$\begin{aligned} t &= t_0 T, \quad z = \delta Z, \quad v = u_0 V, \quad p = \rho_0 u_0^2 P, \\ c &= c^* C, \end{aligned} \tag{20}$$

where t_0 (s) is the characteristic time scale of the process. The length δ denotes the depth of the gaseous layer above the liquid where the principle changes of both the concentration and the velocity occur. The value of u_0 is the characteristic velocity scale. The results is:

$$\begin{aligned} \frac{\partial V}{\partial T} + \frac{u_0 t_0}{\delta} V \frac{\partial V}{\partial Z} &= -\frac{u_0 t_0}{\delta} \frac{\partial P}{\partial Z} + \frac{\vartheta t_0}{\delta^2} \frac{\partial^2 V}{\partial Z^2} - \frac{g \alpha t_0 c^*}{\rho_0 u_0} C, \\ \frac{\partial C}{\partial T} + \frac{u_0 t_0}{\delta} V \frac{\partial C}{\partial Z} &= \frac{D t_0}{\delta^2} \frac{\partial^2 C}{\partial Z^2}; \end{aligned} \tag{21}$$

$$\begin{aligned} T = 0, \quad V = C = 0; \\ Z = 0, \quad V = -\frac{D c^* (1 - \alpha)}{c_0^* \delta u_0} \left(\frac{\partial C}{\partial Z} \right)_{Z=0}, \quad C = 1; \\ Z \rightarrow \infty, \quad V = C = 0. \end{aligned} \tag{22}$$

The existence of the Stephan flow leads to the occurrence of flow inside the gaseous phase whose characteristic velocity is defined by the condition (22):

$$\frac{D c^* (1 - \alpha)}{c_0^* \delta u_0} \sim 1, \quad u_0 = \frac{D c^* (1 - \alpha)}{c_0^* \delta}, \tag{23}$$

i.e. both terms in the boundary conditions of the Stephan flow (22) should have equal orders of magnitude.

If the evaporation rate is limited by the non-stationary diffusion, the parameters of both the non-stationary and the diffusion terms of the diffusion equation of the set (21) should have equal orders of magnitude:

$$\frac{D t_0}{\delta^2} \sim 1, \quad \delta = \sqrt{D t_0} \sim 10^{-2} \text{ m} \quad \text{if } t_0 \sim 10^2 \text{ s}. \tag{24}$$

In this way the characteristic velocity of (23) is

$$u_0 = \frac{c^*}{c_0^*} \sqrt{\frac{D}{t_0}} \sim 10^{-5} \text{ m/s}. \tag{25}$$

Including (24) in the first equation of (21) leads to:

$$\frac{\vartheta t_0}{\delta^2} = S c \sim 1, \quad S c = \frac{\vartheta}{D}. \tag{26}$$

If suggest that the flow is limited by the natural convection, the first equation of (19) becomes:

$$\begin{aligned} \frac{\rho_0 u_0}{g \alpha t_0 c^*} \frac{\partial V}{\partial T} + \frac{\rho_0 u_0^2}{g \alpha \delta c^*} V \frac{\partial V}{\partial Z} &= -\frac{\rho_0 u_0^2}{g \alpha \delta c^*} \frac{\partial P}{\partial Z} + \frac{\vartheta \rho_0 u_0}{\delta^2 g \alpha c^*} \\ &\times \frac{\partial^2 V}{\partial Z^2} - C. \end{aligned} \tag{27}$$

In this particular case the parameters of both last terms of (27) should have equal orders of magnitudes:

$$\frac{\vartheta \rho_0 u_0^0}{\delta^2 g \alpha c^*} \sim 1, \quad \delta = \sqrt{\frac{\vartheta \rho_0}{g \alpha c_0^*} \sqrt{\frac{D}{t_0}}} \sim 10^{-5} \text{ m}. \tag{28}$$

The conditions (27) and (28) indicate that the effects of the Stephan flow and the natural convection occur in different zones of the gaseous layer above the liquid surface. This fact permits to separate these effects if the velocity, pressure and the concentration in the first equation of (17) may be expressed in the form

$$v + \bar{v}, \quad \bar{p}, \quad \bar{c}. \tag{29}$$

Here \bar{v} , \bar{p} and \bar{c} are determined by (19), while (17) with a neglected last two terms of the first equation determine the values of v , p and c . In this way the form of (21) becomes:

$$\begin{aligned} \frac{\partial V}{\partial T} + \beta V \frac{\partial V}{\partial Z} &= S c \frac{\partial^2 V}{\partial Z^2}, \\ \frac{\partial C}{\partial T} + \beta V \frac{\partial C}{\partial Z} &= \frac{\partial^2 C}{\partial Z^2}; \\ T = 0, \quad V = C = 0; \end{aligned} \tag{30}$$

$$\begin{aligned} Z = 0, \quad V = -\frac{\partial C}{\partial Z}, \quad C = 1; \\ Z \rightarrow \infty, \quad V = C = 0, \end{aligned}$$

where β follows directly from (22) and (24)–(26):

$$\beta = (1 - \alpha) \frac{c^*}{c_0^*} \sim 10^{-1}. \tag{31}$$

Obviously Eq. (30) are valid within a broad initial time interval t_0 , when the thickness of the diffusion boundary layer $\delta = (D t_0)^{1/2}$ is lower than of the depth of the gaseous phase l (in the cases studied in [13], $l = 0.257$ m).

7. Asymptotic solution

The solution of (30) may be obtained as a series of the powers of a small parameter β :

$$V = V_0 + \beta V_1, \quad C = C_0 + \beta C_1. \tag{32}$$

Thus, the zero-order approximation is:

$$\begin{aligned} \frac{\partial V_0}{\partial T} &= S c \frac{\partial^2 V_0}{\partial Z^2}, \\ \frac{\partial C_0}{\partial T} &= \frac{\partial^2 C_0}{\partial Z^2}; \\ T = 0, \quad V_0 = C_0 = 0; \\ Z = 0, \quad V_0 &= -\frac{\partial C_0}{\partial Z}, \quad C_0 = 1; \\ Z \rightarrow \infty, \quad V_0 = C_0 = 0. \end{aligned} \tag{33}$$

The solution for C_0 is:

$$C_0 = \operatorname{erfc} \frac{Z}{2\sqrt{T}}. \tag{34}$$

The problem for V_0 is:

$$\begin{aligned} \frac{\partial V_0}{\partial T} &= Sc \frac{\partial^2 V_0}{\partial Z^2}; \\ T = 0, \quad V_0 &= 0; \\ Z = 0, \quad V_0 &= \varphi(T) = \frac{1}{\sqrt{\pi T}}; \\ Z \rightarrow \infty, \quad V_0 &= 0. \end{aligned} \tag{35}$$

The solution of (35) may be obtained by the Green functions [17]:

$$\begin{aligned} V_0 &= \varphi(T) \exp\left(-\frac{Z^2}{4ScT}\right) - \int_0^T \frac{\varphi(\tau) + 2\tau\varphi'(\tau)}{\sqrt{\tau}} \\ &\times \left[\int_0^{(\tau Z)/(4ScT\tau(T-\tau))^{1/2}} \exp(-u^2) du \right] d\tau. \end{aligned} \tag{36}$$

This permits to determine at $\varphi = (\pi T)^{-1/2}$ directly the value of V_0

$$V_0 = \frac{\exp\left(-\frac{Z^2}{4ScT}\right)}{\sqrt{\pi T}}. \tag{37}$$

The problem formulation for C_1 follows from (30):

$$\begin{aligned} \frac{\partial C_1}{\partial T} - \frac{\partial^2 C_1}{\partial Z^2} &= V_0 \frac{\partial C_0}{\partial Z} = -\frac{\exp\left[-\frac{Z^2}{4T}\left(1 + \frac{1}{Sc}\right)\right]}{\pi T}; \\ T = 0, \quad C_1 &= 0; \\ Z = 0, \quad C_1 &= 0; \\ Z \rightarrow \infty, \quad C_1 &= 0. \end{aligned} \tag{38}$$

The solution of (38) obtain through Green functions [17] is

$$\begin{aligned} C_1 &= \frac{\exp\left(-\frac{Z^2}{4T}\right)}{2\pi\sqrt{\pi}} \int_0^T \frac{1}{\tau\sqrt{T-\tau}} \\ &\times \int_0^\infty \left\{ \exp\left[-\frac{\xi(T-Z\tau)^2}{4T\tau(T-\tau)}\right] \right. \\ &\quad \left. - \exp\left[-\frac{(\xi T + Z\tau)^2}{4T\tau(T-\tau)}\right] \right\} \\ &\times \exp\left(-\frac{\xi^2}{4Sc\tau}\right) d\xi d\tau. \end{aligned} \tag{39}$$

From (39) it follows that:

$$\left(\frac{\partial C_1}{\partial Z}\right)_{Z=0} = -\frac{2}{\pi} \sqrt{\frac{Sc}{\pi T}} \operatorname{arctg}(Sc^{-1/2}). \tag{40}$$

The amount of the evaporated liquid Q (kg/m²) yields from (15), (32), (34) and (40):

$$\begin{aligned} Q &= \int_0^{t_0} J dt \\ &= -c^* \frac{\rho_0^*}{c_0^*} \sqrt{Dt_0} \int_0^1 \left[\left(\frac{\partial C_0}{\partial Z}\right)_{Z=0} \right. \\ &\quad \left. + \frac{c^*(1-\alpha)}{c_0^*} \left(\frac{\partial C_1}{\partial Z}\right)_{Z=0} \right] dT \\ &= A\sqrt{t_0}, \end{aligned} \tag{41}$$

where

$$A = 2c^* \frac{\rho_0^*}{c_0^*} \left(\sqrt{\frac{D}{\pi}} + \frac{2c^*(1-\alpha)}{\pi c_0^*} \sqrt{\frac{\vartheta}{\pi}} \operatorname{arctg}(Sc^{-1/2}) \right). \tag{42}$$

The expression (42) permit to calculate the parameter A for various systems, summarized in Table 1. The values are close to the those of A_D , but in the dominating situations are quite different from the values of A_e . This indicates that the existence of an additional convective transport, that could be provoked by of a loss of stability of the system. Thus, the small disturbances grow up to the establishment of stable amplitudes and the dissipative structures formed have greater rate of the transport processes.

8. Conclusions

The results developed here and their comparisons with the experimental data of [13] indicate that during the evaporation of low volatile liquids (H₂O, CH₃OH, C₂H₅OH, *i*-C₃H₇OH) in gaseous media of indifferent gases (N₂, He, Ar) and externally thermostatic conditions the temperature of the liquid surface is practically constant. This shows that there is no possibility to create thermocapillary flow as suggested in [13].

Upon such conditions the mass transfer in the gaseous phase limits the evaporation rate, where the convective contribution depends on the Stephan flow.

The comparison between the theoretical and the experimental results shows that the existence of an additional convective transport probably induced not only the Stephan flow but of a stability loss of the system.

The experimental data of [13] shows that at large characteristic time ($\delta = (Dt_0)^{1/2} \geq l = 0.257$ m), $Q = \text{const}$, that may be explained with the non-stationary diffusion between both surfaces. Thus the boundary condition in (17), $z \rightarrow \infty$, $c = 0$ must be replaced by $z = l$, $\partial c/\partial z = 0$. Upon conditions imposed by small characteristic times (where $\delta < l$) the determination of the additional convective transport (induced by the system instability and the onset of a natural convection) is required.

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